12.2 – Law of Sines

Learning Objectives:

1. Use trigonometric ratios for angle measures greater than 90.
2. Solve problems using the Law of Sines to find missing side lengths and angle measures of nonright triangles.
3. Use the Law of Sines to investigate the ambiguous case.

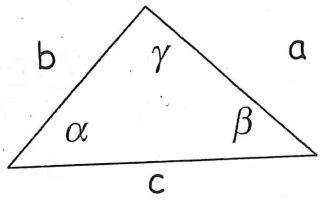
Recall: Right triangles can be solved using SOH CAH TOA

Oblique triangles (do not contain a right angle, either acute or obtuse) can be solved given 3 of the 6 measurements.

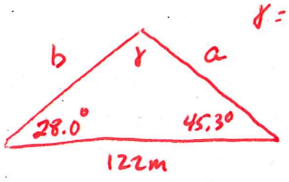
Use Law of Sines given:

1) Two angles and any side (ASA or AAS)
2) Two sides and an angle opposite one of them (SSA – Ambiguous Case- could have 0, 1 or 2 triangles)

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$



Example 1: Solve the triangle with the given info: $\alpha = 28.0°, \beta = 45.3°, c = 122m.$



$\gamma = 180° - (28.0 + 45.3)$

$180° - 73.3$

$\gamma = 106.7°$

$\frac{\sin 106.7}{122} = \frac{\sin 45.3°}{b}$

$b \sin 106.7 = 122 \sin 45.3°$

$\frac{b \sin 106.7}{\sin 106.7°} = \frac{122 \sin 45.3°}{\sin 106.7°}$

$b = 90.536m.$

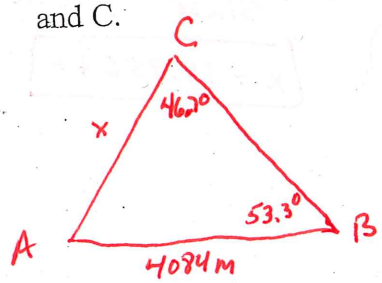
$\frac{\sin 106.7}{122} = \frac{\sin 28.0}{a}$

$a \sin 106.7 = 122 \sin 28°$

$\frac{a \sin 106.7}{\sin 106.7} = \frac{122 \sin 28°}{\sin 106.7}$

$a = 59.798\,m$

Example 2: Towers A, B, and C are located in a national forest. From Tower B, the angle between Towers A and C is 53.3°, and from Tower C, the angle between Towers A and B is 46.7°. The distance between Towers A and B is 4084 m. A lake between Towers A and C makes it difficult to measure distance between them directly? What is the distance between Towers A and C.

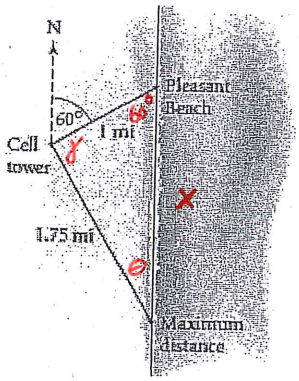


$\frac{\sin 46.7°}{4084} = \frac{\sin 53.3°}{x}$

$\frac{x \sin 46.7°}{\sin 46.7°} = \frac{4084 \sin 53.3°}{\sin 46.7°}$

$x = 4499.278$

Example: 3 The Hear Me Now Phone Company plans to build a cell tower to serve the needs of Pleasant Beach and the beachfront. It decides to locate the cell tower so that Pleasant Beach is 1 mile away at an angle of 60° clockwise from the north. The range of the signal from the cell tower is 1.75 miles. The beachfront runs north to south. How far south of Pleasant Beach will customers be able to use their cell phones?



SKIP

$\frac{\sin 60^\circ}{1.75} = \frac{\sin\theta}{1}$

$\frac{1.75\sin\theta}{1.75} = \frac{\sin 60^\circ}{1.75}$

$\sin\theta = \frac{\sin 60}{1.75}$

$\theta = \sin^{-1}\left(\frac{\sin 60^\circ}{1.75}\right)$

$\theta = 29.66128776$

$\gamma = 180 - (60 + 29.66128776)$

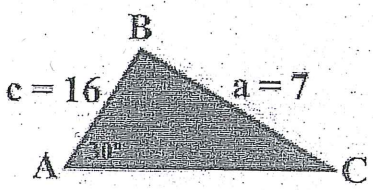
$\gamma = 90.33871224$

$\frac{\sin 60^\circ}{1.75} = \frac{\sin 90.3387}{x}$

$\frac{x\sin 60^\circ}{\sin 60^\circ} = \frac{1.75\sin 90.3387}{\sin 60^\circ}$

$x = 2.021$ m.

Example 4: Given the following triangle, how many distinct triangles can be drawn? Explain.



SKIP

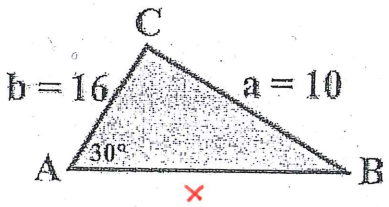
$\frac{\sin 30^\circ}{7} = \frac{\sin C}{16}$

$\frac{7\sin C}{7} = \frac{16\sin 30^\circ}{7}$

$\sin C = 1.142857$

IMPOSSIBLE $\sin C$ must be less than 1.

Example 5: You are building support beams for a building. These beams will each be attached to the floor and will meet to form a triangle. You are told you will have beams that are 16 feet long and 10 feet long. The angle created between the 16 foot beam and the ground is 30°. How far apart should they be placed on the floor?



$\frac{\sin 30^\circ}{10} = \frac{\sin B}{16}$

$\frac{10\sin B}{10} = \frac{16\sin 30^\circ}{10}$

$\sin B = \frac{16\sin 30^\circ}{10}$

$B = \sin^{-1}\left(\frac{16\sin 30^\circ}{10}\right)$

$B = 53.130^\circ$

$C = 180 - (30 + 53.130)$

$C = 96.86989765^\circ$

$\frac{\sin 30^\circ}{10} = \frac{\sin 96.86989765^\circ}{x}$

$\frac{x\sin 30^\circ}{\sin 30^\circ} = \frac{10\sin 96.86989765^\circ}{\sin 30^\circ}$

$x = 19.856$ ft